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Guidance Algorithm Design: A Nonlinear Inverse Approach

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The design of a missile guidance algorithm valid for any initial missile and target orientation is described. The limitations of proportional navigation- (PN-) or line-of-sight rate-based guidance for this problem are pointed out. Design heuristics are explained, and general guidelines for the synthesis of guidance algorithms are stated. It is shown that the design of an algorithm valid for any initial missile and target orientation leads to a nonlinear inverse problem. Standard geometric methods for solving nonlinear inverse problems are adapted to derive a new class of algorithms based on the relative heading error angle (RHEA). Examples of RHEA algorithms are then discussed and compared with PN guidance for different engagement scenarios.

Nomenclature

a_m	= missile acceleration
a_t	= target acceleration
V_m	= missile speed
V_t	= target speed
X_m, Y_m	= dimensional missile coordinates
X_t, Y_t	= dimensional target coordinates
x_m, y_m	= nondimensional missile coordinates
x_t, y_t	= nondimensional target coordinates
α_m	= nondimensional missile acceleration
α_t	= nondimensional target acceleration
θ	= line of sight (LOS) angle
θ'	= LOS rate
ν	= target to missile velocity ratio
ρ	= nondimensional distance to go
τ	= nondimensional time
χ_m	= missile heading angle
χ_t	= target heading angle
ψ	= relative heading error angle

I. Introduction

THE design of guidance algorithms may be defined loosely as the art of finding the correct acceleration commands needed to move between two given points. Many different techniques have been suggested for the design of guidance algorithms. These range from the earliest guidance algorithms derived from physical insight, for example, pursuit, proportional navigation (PN), and their variants[1–4] to those derived from a systematic application of mathematical techniques.[5–10] References 5–10 are included as a representative sample of more recent works. Recent trends in guidance algorithm design may be classified into two main approaches: 1) the more established approach using optimization theory[5–8] and 2) the more recent approach using nonlinear geometric methods.[8–10] A combination of both techniques has also been attempted.[8] Guidance algorithms designed using optimization techniques seek to minimize some specified objective such as the total acceleration required or the time needed for an intercept. Although theoretically appealing, the resulting guidance algorithms are often difficult to implement because the acceleration commands must be solved in real time with full state measurements. Nevertheless, these optimal algorithms do provide useful design benchmarks. The newer nonlinear geometric approach is applicable to systems that are linear in the acceleration and aims to construct guidance algorithms with specified desired, usually linear, dynamics, i.e., solve an inverse problem. Unfortunately guidance laws derived by a naive application of nonlinear geometric methods can suffer from serious flaws. In Ref. 10 such a guidance algorithm was found to be singular when the missile heading is correctly aligned with the target for interception. As a result, the algorithm was supplemented with traditional PN to ensure an intercept.

In contrast to the methods mentioned, we adopt a different approach to the design problem by first examining the underlying kinematics of the intercept problem. In particular, we are concerned with the design of a guidance algorithm valid for any initial missile and target orientation. This problem is related to the design of missiles with all-aspect capabilities. We explain why traditional PN- or line-of-sight- (LOS-) rate-based guidance algorithms are not suitable for this problem. We then show that a guidance algorithm valid for any initial missile and target orientation can be derived if we avoid the usual approach of using geometric methods as a means for linearizing the dynamics.

The rest of the paper is organized as follows: Section II defines the geometry of our guidance problem and provides a short introduction to the solution of an inverse problem using nonlinear geometric methods. Section III states some useful heuristics for guidance algorithm design and modifies the standard nonlinear geometric approach for solving a nonlinear inverse problem. A general class of relative heading error angle (RHEA) guidance algorithms is then derived. Finally, in Sec. IV simulation results are presented to compare the performance of a specific RHEA algorithm with PN guidance.

II. Background and Problem Formulation

The geometry for the problem is defined in Fig. 1. The acceleration vectors for both missile and target are orthogonal to their respective velocity vectors, as is typical of aerodynamic controlled missiles and targets. This also implies that the applied acceleration changes the direction but not the magnitude of the velocity vector. Our main aim is to design a guidance algorithm valid for any initial missile χ_m and target χ_t heading angles. Using a fixed Cartesian reference frame, the kinematic equations for a missile are

$$X_m' = V_m \cos\chi_m, \qquad X_m(0) = 0 \tag{1a}$$

$$Y_m' = V_m \sin\chi_m, \qquad Y_m(0) = 0 \tag{2a}$$

$$\chi_m' = a_m/V_m, \qquad \chi_m(0) = \chi_m(0) \tag{3a}$$

and for a target are

$$X_t' = V_t \cos\chi_t, \qquad X_t(0) = R_0 \tag{4a}$$

$$Y_t' = V_t \sin\chi_t, \qquad Y_t(0) = 0 \tag{5a}$$

$$\chi_t' = a_t/V_t, \qquad \chi_t(0) = \chi_t(0) \tag{6a}$$

Without any loss of generality we have chosen the origin of the Cartesian reference frame to coincide with the missile initial position and the x axis aligned with the initial LOS. For convenience,

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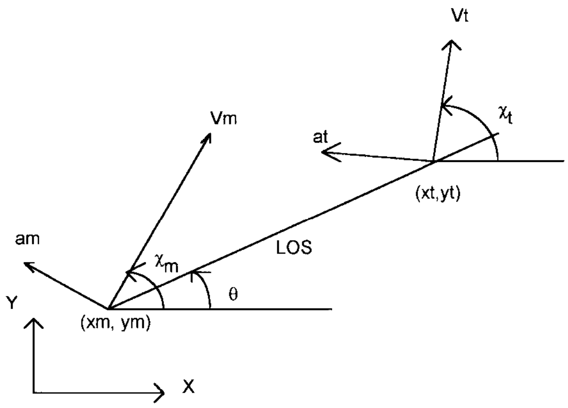

Fig. 1 Problem definition.

Eqs. (1–6) can be rewritten in nondimensional form by scaling all distances by R_0 and velocities by V_m. For a missile, this leads to

$$x_m' = \cos\chi_m, \qquad x_m(0) = 0 \tag{1b}$$

$$y_m' = \sin\chi_m, \qquad y_m(0) = 0 \tag{2b}$$

$$\chi_m' = \alpha_m, \qquad \chi_m(0) = \chi_m(0) \tag{3b}$$

and for a target this leads to

$$x_t' = \nu\cos\chi_t, \qquad x_t(0) = 1 \tag{4b}$$

$$y_t' = \nu\sin\chi_t, \qquad y_t(0) = 0 \tag{5b}$$

$$\chi_t' = \alpha_t/\nu, \qquad \chi_t(0) = \chi_t(0) \tag{6b}$$

with the understanding that the prime now denotes differentiation with respect to the new nondimensional time variable $\tau = R_0/V_m t$ and the velocity ratio parameter $\nu = (V_t/V_m)$ is always nonnegative and is less than 1. For the intercept problem, it is the relative distance of the target from the missile that matters; hence, we define the relative coordinates

$$x_r' = (x_t - x_m)' = \nu\cos\chi_t - \cos\chi_m \tag{7}$$

$$y_r' = (y_t - y_m)' = \nu\sin\chi_t - \sin\chi_m \tag{8}$$

and letting $x_r = \rho\cos\theta$ and $y_r = \rho\sin\theta$ in Eqs. (7) and (8) we obtain the governing equations for ρ, the distance from the missile to the target and the LOS angle θ,

$$\rho' = -\nu\cos(\chi_t - \theta) + \cos(\chi_m - \theta) \tag{9}$$

$$\rho\theta' = \nu\sin(\chi_t - \theta) - \sin(\chi_m - \theta) \tag{10}$$

Equations (9) and (10) can also be obtained by a more elegant but perhaps less obvious approach using a frame of reference rotating with the LOS.

At this point it is necessary to highlight the key features of the inverse problem studied in Refs. 9 and 10, where the main aim is to improve the performance of PN guidance using differential geometric methods. For a successful interception, ρ has to go to zero, and the inverse problem requires us to find the correct missile acceleration with this desired property for ρ. The standard solution is to differentiate Eq. (9) once to obtain an expression containing the missile acceleration explicitly

$$\rho'' = -\rho(\theta')^2 + \alpha_t\sin(\chi_t - \theta) - \alpha_m\sin(\chi_m - \theta) \tag{11}$$

The required missile acceleration is then obtained from Eq. (11) by setting

$$\alpha_m = \frac{-\rho'' - \rho(\theta')^2 + \alpha_t\sin(\chi_t - \theta)}{\sin(\chi_m - \theta)} \tag{12}$$

where the desired behavior of ρ is chosen to be the output from any asymptotically stable linear system. This is essentially the method in Refs. 9 and 10, where a slightly longer derivation in Cartesian coordinates was used. The major problem with this algorithm is the $\sin(\chi_m - \theta)$ term in the denominator of Eq. (12). For a direct head-on or tail chase interception, the missile heading will coincide with the LOS, which means that $\sin(\chi_m - \theta) = 0$ or infinite acceleration will be required. Physically, this is unrealistic because one expects the exact opposite, i.e., no acceleration should be required because the missile is already on course for an interception. As such, this guidance algorithm was modified so that for small $(\chi_m - \theta)$, a switch to PN guidance is used to ensure an intercept.

Though the preceding algorithm has its limitations, it is still interesting because it represents an attempt to modify PN guidance for any initial missile and target orientation. PN guidance is a simple and well-tested algorithm. The key idea is that if a missile is on an intercept course the LOS will not rotate. Hence, the required acceleration is taken to be proportional to the LOS rate. It is also well known that PN guidance minimizes the required acceleration provided the missile velocity vector is close to the LOS initially, i.e., small initial misalignment. For increasing initial misalignment, the performance of PN guidance degrades. In fact, for a target moving directly away from the missile in a straight line, PN guidance fails completely because the LOS rate is zero leading to no acceleration command. Physically for this case, one expects that maximum acceleration should be used to turn the missile back toward the target.

The reason PN and the algorithm in Ref. 10 fails to work for any initial missile and target orientation is that for an interception to occur it is not the LOS rate or the distance-to-go that should be selected as the output to be driven to zero. Rather, what really matters is the alignment of the relative velocity vector with the LOS, and the variable that must be driven to zero is the angle between the relative velocity vector and the LOS, which we refer to as the RHEA. (This should not be confused with pursuit guidance, which is based on aligning the missile velocity vector with the LOS.) With reference to Fig. 1, we define the vector $\boldsymbol{r} = \{\cos\theta, \sin\theta\}$ along the LOS and the relative velocity vector $\boldsymbol{V_r} = \{\nu\cos\chi_t - \cos\chi_m, \nu\sin\chi_t - \sin\chi_m\}$. Taking the dot product yields

$$\begin{aligned} V_r\cos\psi &= -\boldsymbol{r}\cdot\boldsymbol{V_r} \\ &= \nu\cos(\chi_t - \theta) - \cos(\chi_m - \theta) \\ &= -\rho' \end{aligned} \tag{13}$$

[compare the last equality with Eq. (9)]. From Fig. 2, we observe that there are two possible cases, which depend on whether $\boldsymbol{V_r}$ tends to increase or decrease the LOS angle. In the first case, where θ' is positive, we define ψ to lie in the range 0–π. For the second case, where θ' is negative, we take ψ to lie in the range $-\pi$–0. This convention allows us to define, using the cross product $\boldsymbol{r}\times\boldsymbol{V_r}$, a complementary relationship

$$\begin{aligned} V_r\sin\psi &= \nu\sin(\chi_t - \theta) - \sin(\chi_m - \theta) \\ &= \rho\theta' \end{aligned} \tag{14}$$

[compare the last equality with Eq. (10)]. Thus, we observe that the RHEA directly controls both the distance-to-go as well as the

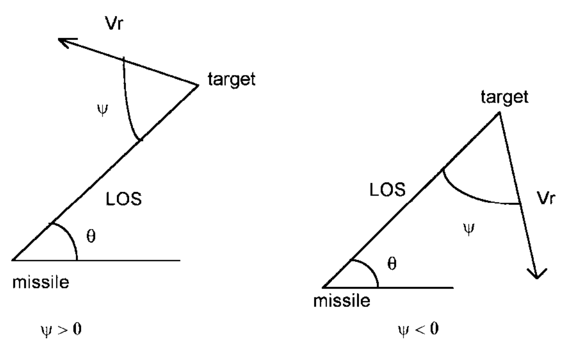

Fig. 2 Convention for the RHEA.

LOS rate, and driving ψ to zero ensures that both θ' and ρ will go to zero, an important property omitted from previous studies. Finally we need to derive the governing differential equation for ψ. Differentiating Eqs. (13) and (14) leads to

$$\cos\psi V_r' - \sin\psi V_r \psi' = \alpha_t \sin(\chi_t - \theta) - \alpha_m \sin(\chi_m - \theta) - \theta' V_r \sin\psi \tag{15}$$

$$\sin\psi V_r' + \cos\psi V_r \psi' = \alpha_t \cos(\chi_t - \theta) - \alpha_m \cos(\chi_m - \theta) + \theta' V_r \cos\psi \tag{16}$$

Solving for V_r' and $V_r\psi'$ we obtain

$$V_r' = \alpha_t \sin(\chi_t - \theta + \psi) - \alpha_m \sin(\chi_m - \theta + \psi) \tag{17}$$

$$V_r\psi' = \alpha_t \cos(\chi_t - \theta + \psi) - \alpha_m \cos(\chi_m - \theta + \psi) + V_r\theta' \tag{18}$$

Based on these results we are now almost ready to begin the algorithm design.

III. Algorithm Design Heuristics

Before beginning the algorithm design it is advantageous to consider the kinematics involved in the intercept problem. Failure to appreciate the underlying kinematics can often lead to some rather meaningless derivations and results. Any guidance algorithm, no matter how complicated, must satisfy three basic rules. Intuitively, if the missile is on track for an interception, then the algorithm should not generate an acceleration command. We refer to this as the first basic rule for guidance algorithm design. Another important observation is that for a vehicle moving away from the target ($\psi(0) < -\pi/2$ or $\psi(0) > \pi/2$) the distance-to-go ρ must first increase as acceleration is applied to steer the vehicle back toward the target. Noting that $\rho^2 = \boldsymbol{r}\cdot\boldsymbol{r}$, the maximum distance-to-go occurs when $\mathrm{d}(\rho^2)/\mathrm{d}\tau = 0$ or when $(\boldsymbol{r}\cdot\boldsymbol{V}_r) = 0$. This implies that ρ is maximum when the RHEA is $\pm\pi/2$. This is the second basic rule, and it holds for any interception scenario, independent of the guidance algorithm. In other words, for the inverse problem, one cannot simply specify ρ to follow any desired dynamics. The desired dynamics for ρ must be kinematically reasonable. This is another point overlooked by previous studies. Finally, the third observation is that for an intercept one should align the relative velocity vector with the LOS. Hence, the algorithm must ensure that the RHEA ψ goes to zero. Ideally, ψ should go to zero monotonically with time. The three basic rules for guidance algorithm design are summarized as follows.

Rule 1:

$$\alpha_m(\psi = 0) = 0$$

Rule 2:

$$\frac{\mathrm{d}\rho}{\mathrm{d}\psi} = 0 \qquad \text{and} \qquad \frac{\mathrm{d}^2\rho}{\mathrm{d}\psi^2} < 0 \qquad \text{at} \qquad \psi = \pm\pi/2$$

Rule 3:

$$\frac{\mathrm{d}\psi}{\mathrm{d}\tau} < 0 \qquad \text{for} \qquad 0 < \psi < \psi(\tau = 0) < \pi$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}\tau} > 0 \qquad \text{for} \qquad -\pi < \psi(\tau = 0) < \psi < 0$$

An important consequence of rule 2 is that for a guidance law to be valid for all RHEA, one can write the distance ρ as a function of ψ but not vice versa. This will be significant in the subsequent derivation. Graphically, the behavior of ρ as a function of ψ should be similar to that shown in Fig. 3 for different initial RHEA. Note that in Fig. 3, when the RHEA is 0, the distance-to-go need not be zero. Physically, this means that the vehicle is now on track, and in the absence of any target maneuvers, the missile will continue to move toward the target in a straight line. With these points in mind,

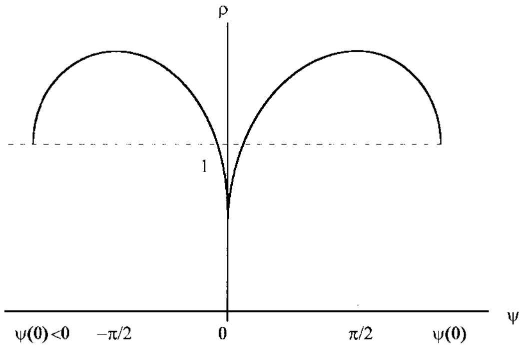

Fig. 3 Qualitative behavior of distance-to-go vs RHEA.

we can now proceed to find the required acceleration command by dividing Eq. (9) by Eq. (18) to obtain

$$\frac{\mathrm{d}\rho}{\mathrm{d}\psi} = \frac{-V_r^2\cos\psi}{\alpha_t\cos(\chi_t - \theta + \psi) - \alpha_m\cos(\chi_m - \theta + \psi) + V_r\theta'} \tag{19}$$

We need to use ψ as the independent variable because, as mentioned earlier, ρ is a function of ψ and not vice versa. Equation (19) is nonlinear in the acceleration. Hence, standard geometric methods[11] for linearizing the system do not apply. Instead of specifying the desired dynamics for ρ, we let the desired dynamics for ψ to be of the form

$$\frac{\mathrm{d}\psi}{\mathrm{d}\tau} = -V_r g(\psi) \tag{20}$$

where $g(\psi)$ is a function to be determined. Applying the three design rules, the function g should possess the following properties.

Property 1:

$$g(\psi) = -g(-\psi)$$

i.e., g is an odd function and, hence, $g(0) = 0$.

Property 2:

$$g(\psi) > 0 \qquad \text{for} \qquad 0 < \psi \le \pi$$
$$g(\psi) < 0 \qquad \text{for} \qquad -\pi \le \psi < 0$$

Together these properties ensure that ψ will go to zero from any initial RHEA ψ_0. A more rigorous proof may be performed using the Lyapunov function $V(\psi) = \frac{1}{2}\psi^2$,

$$\frac{\mathrm{d}V}{\mathrm{d}\tau} = \psi\psi' = -V_r\psi g(\psi) \tag{21}$$

Because V_r is the magnitude of the relative velocity, it is always positive. By properties 1 and 2, the term $\psi g(\psi)$ is an even function of ψ and is nonnegative. Hence, V is a decreasing function of time and must tend to its minimum value at $\psi = 0$. Choosing this desired dynamics for ψ requires that the missile acceleration takes the form [from Eq. (18)]

$$\alpha_m = \frac{\alpha_t\cos(\chi_t - \theta + \psi) + V_r^2 g(\psi) + V_r\theta'}{\cos(\chi_m - \theta + \psi)} \tag{22}$$

The form for the acceleration command is similar to that derived in Ref. 10, but an important difference in this approach is that the guidance algorithm is never singular. Taking the denominator in Eq. (22) and using Eqs. (13) and (14),

$$\cos(\psi_m - \theta + \psi) = \cos(\chi_m - \theta)\cos(\psi) - \sin(\chi_m - \theta)\sin(\psi) = \frac{1 - \nu\cos(\chi_t - \chi_m)}{V_r} \tag{23}$$

Because the velocity ratio ν is always less than 1 and $\cos(\chi_t - \chi_m)$ always lies between ± 1, the following inequality holds:

$$0 < [1 - \nu]/V_r \le \cos(\psi_m - \theta + \psi) \le [1 + \nu]/V_r \tag{24}$$

In other words, the algorithm is never singular, and it holds for any missile and target orientation. Substituting Eq. (22) in Eq. (19), the distance-to-go must behave as

$$\frac{\mathrm{d}\rho}{\mathrm{d}\psi} = \frac{\cos\psi}{g(\psi)} \tag{25}$$

Integrating Eq. (25) yields

$$\rho(0) = 1 - \int_0^{\psi_0} \frac{\cos\psi}{g(\psi)}\,\mathrm{d}\psi = 1 - I(\psi) \tag{26}$$

Because $\rho(0)$ is the distance at which the RHEA becomes zero, one expects this distance to be bounded and nonnegative. Although the integrand in Eq. (26) may be singular at $\psi = 0$, what matters is that the integral must remain finite. Thus, the design of the guidance algorithm for any missile and target orientation is reduced to the problem of finding a suitable function g. The missile guidance algorithm design can be restated as follows.

Problem: Missile guidance algorithm design. Find an odd function $g(\psi)$ such that g is positive for positive ψ and the integral $I(\psi)$ in Eq. (26) is bounded and is never greater than 1.

For such functions, Eq. (22) defines a class of RHEA guidance algorithms. One possible class of functions can be constructed as follows. Note that for $-\pi \leq \psi \leq \pi$,

$$\frac{-1}{|\psi|^{1/n}} \leq \frac{\cos\psi}{|\psi|^{1/n}} \leq \frac{1}{|\psi|^{1/n}} \tag{27}$$

Thus, for any initial RHEA, ψ_0 and $n > 1$, the integral $I(\psi)$ is always bounded for functions $g(\psi)$ of the form

$$g(\psi) = K\,\mathrm{sign}(\psi)|\psi|^{1/n} \qquad K > 0 \tag{28}$$

where K is the algorithm gain and $\mathrm{sign}(\psi) = -1$ if $\psi < 0$ and is 1 otherwise to ensure that $g(\psi)$ is an odd function. This implies that there exists a class of nth root RHEA guidance algorithms defined as follows:

$$\alpha_m(\psi) = \frac{\alpha_t \cos(\psi_t - \theta + \psi) + K V_r^2\,\mathrm{sign}(\psi)|\psi|^{1/n} + V_r\theta'}{\cos(\psi_m - \theta + \psi)} \qquad \text{for} \quad n > 1 \tag{29}$$

The algorithm gain K is chosen so that $\rho(\psi = 0) \geq 0$. From Eq. (26), for the case $n = 2$, the minimum guidance gain is given by

$$K_{\min} = \int_0^{\pi/2} \frac{\cos\psi}{\sqrt{\psi}}\,\mathrm{d}\psi = 2\int_0^{\sqrt{\pi/2}} \cos t^2\,\mathrm{d}t \tag{30}$$

which is related to a Fresnel integral[12] and $K_{\min} \approx 1.995$. This suggests that for any initial RHEA, a minimum gain of 2 should be adequate for the square root RHEA algorithm. In the next section we will examine the performance of this particular RHEA algorithm.

IV. Simulation Results

In this section the performance of the square root RHEA algorithm is compared with a PN algorithm. In nondimensional terms the simulation parameters are velocity ratio $\nu = 0.2$, missile initial heading $\chi_m(0) = -2.9$ rad, target initial heading $\chi_t(0) = 0.0$ rad, missile initial coordinates $= \{0, 0\}$, and target initial coordinates $= \{1, 0\}$. These parameters correspond to an initial RHEA close to 180 deg so that the PN algorithm does not fail because of zero LOS rate. To check the robustness of the algorithm we have included command saturation and a first-order lag in the missile acceleration. A nondimensional acceleration limit of ± 2 is used with a nondimensional time lag of 0.1. These values are purposely chosen to be worse than the values for a typical air-to-air missile. For a more realistic simulation, the RHEA is computed on-line based on missile and target position and velocities instead of integrating Eqs. (17) and (18) directly. Figure 4 shows the trajectory and missile acceleration (nondimensional) for the PN algorithm. The PN guidance gain is set to the optimal value of 3, as shown in Ref. 13. Figure 5 shows the corresponding results for the RHEA algorithm with guidance gain $K = 2$. It can be observed that the PN algorithm is unable to utilize the maximum allowable acceleration because of the large initial RHEA (and, hence, low-LOS rates) at the start, and it takes about twice as long to intercept the target. For an initial RHEA of 180 deg, the PN algorithm would have failed completely. The RHEA algorithm was also compared with PN algorithms supplemented with various turning strategies for large initial misalignment. One of the better turning strategies explored is based on the RHEA, and this algorithm applies maximum acceleration until the RHEA decreases

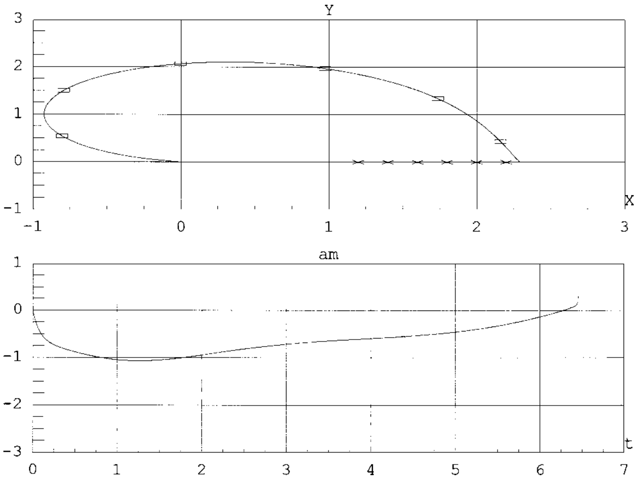

Fig. 4 Trajectory and acceleration vs time plots for PN guidance: ▭, missile, and ×, target position at unit time intervals.

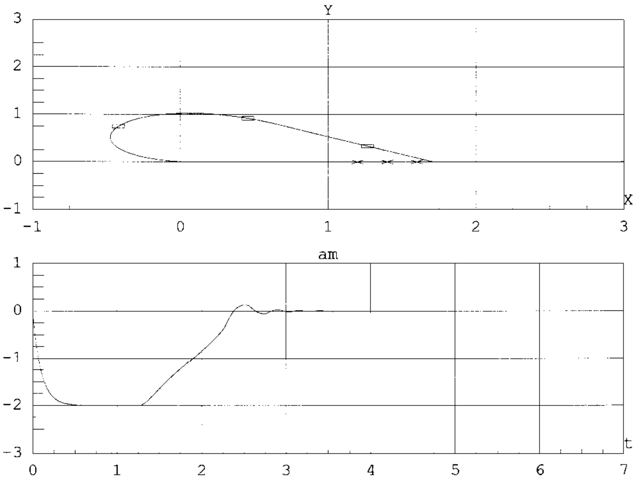

Fig. 5 Trajectory and acceleration vs time plots for RHEA guidance: ▭, missile, and ×, target position at unit time intervals.

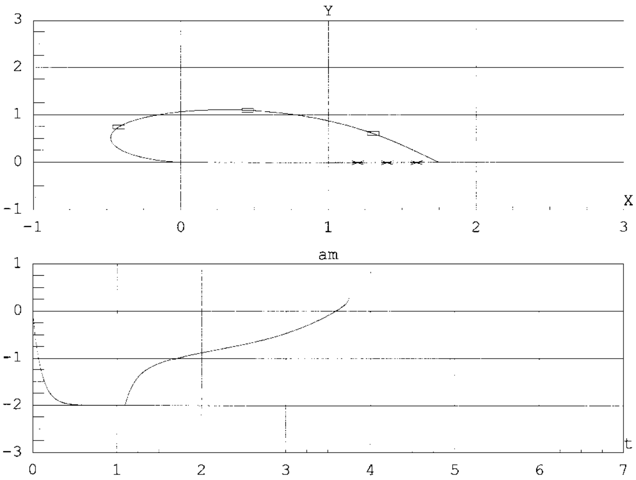

Fig. 6 Trajectory and acceleration vs time plots for modified PN guidance: ▭, missile, and ×, target position at unit time intervals.

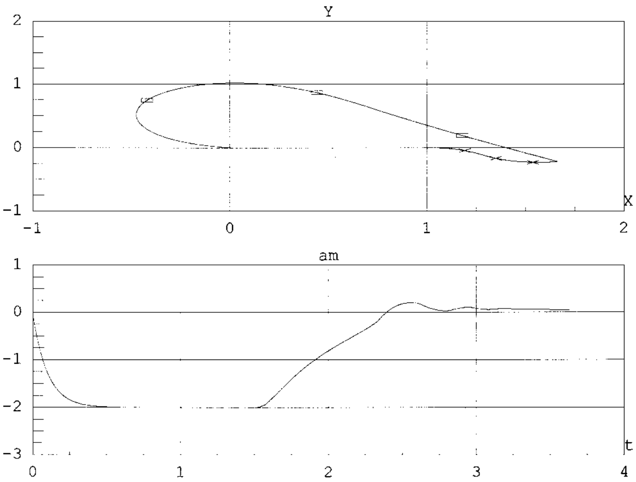

Fig. 7 Trajectory and acceleration vs time plots for RHEA guidance–maneuvering target: ▭, missile, and ×, target position at unit time intervals.

to within ±90 deg and then it switches over to PN guidance. Physically, this means that the algorithm tries to correct the misalignment as quickly as possible. In Fig. 6, the trajectory and acceleration for this modified PN algorithm is shown for the same initial conditions. Comparing with Fig. 5, the time to intercept for both algorithms are both close to 3.5 nondimensional time units, but the RHEA algorithm clearly uses less acceleration and places the missile in an almost direct intercept course (no acceleration) at the end of the turn. Simulations for a maneuvering target were also carried out. In Fig. 7, the results for a target executing a bank-to-bank turn $[\alpha_t = 0.1\cos(\tau)]$ is shown. In this case the initial RHEA is chosen to be exactly 180 deg, and the RHEA algorithm is capable of steering the missile back toward the target. Similar results were obtained for other initial missile and target orientations, and the algorithm was found to be robust to acceleration saturation and lags.

V. Conclusions

The key results of this study are as follows.

1) A different perspective for designing guidance algorithms is stated, and it is shown that for successful target interception, it is sufficient to drive the angle between the relative velocity vector and the LOS or the RHEA to zero.

2) The use of the RHEA as the output, as opposed to more intuitive quantities such as the distance-to-go or LOS rate, leads to a nonsingular solution of the nonlinear inverse guidance algorithm design problem. The drawback is that the resulting equations are nonlinear in the output and cannot be solved using standard differential geometric methods. In this study an appreciation of the physical problem plays an important part in deriving the solution.

3) A general class of RHEA-based algorithms are defined. An important consequence of using the RHEA is that these algorithms are valid for any initial missile and target orientation and do not require additional ad hoc modifications.

4) An explicit example for constructing a specific RHEA algorithm from the general class is stated. Simulation results show that the resulting square root RHEA algorithm is robust to acceleration saturation and lags and compare favorably with PN and modified-PN type algorithms in terms of required acceleration and time to intercept.

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